Lesson 10
NAME:

Graphing Reciprocal Functions

Start by logging on to the Digital Toolbox and navigating to the Online Lesson for instructions.

Objectives

- Determine the vertical and horizontal asymptotes for a rational function.
- Determine the transformations of a graph from the rational parent function.
- Graph/Sketch a rational function in the form of: $y = \frac{a}{x-h} + k$
- State the domain and range for a rational function.

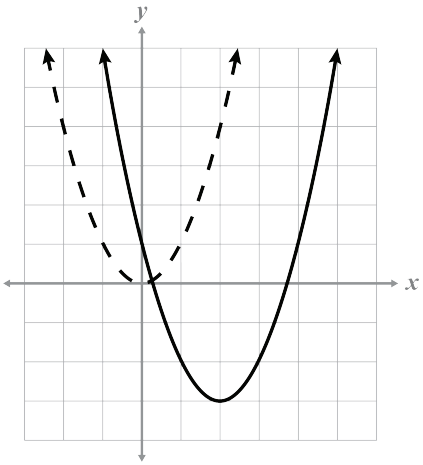
Why?

Knowing what graphs of reciprocal functions look like and how they move will help you make connections when making graphs and transformations in this level. In later levels of math, it will help you to determine if a graph is continuous or discontinuous and discuss end behaviors.

Warm Up

Describe the transformation of a parabola in the form $y = a(x-h)^2 + k$ from the quadratic parent graph $y = x^2$.

1)



2) $y = -(x+4)^2 + 8$

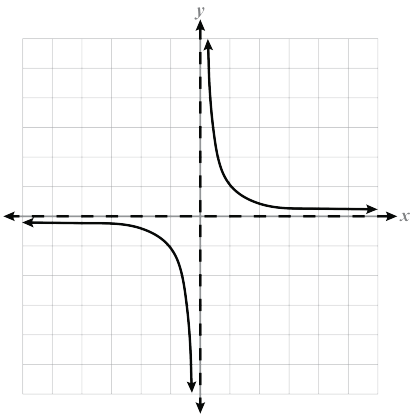
To continue, return to the Online Lesson.

Explore

Reciprocal Functions

Fill in the guided notes as you watch the video in the Online Lesson.

- A reciprocal function is a simple rational function in the form:
- The simplest form of a reciprocal function is its _____ function: $f(x) = \frac{1}{x}, x \neq 0$
- A reciprocal function forms a _____ on the coordinate plane.
- Hyperbolas can have asymptotes that are _____.
  - An _____ is a line that a graph (or curve) approaches (but does not touch) as it moves towards infinity or negative infinity.
  - The asymptote is drawn on a graph using a _____ line. It is used to help create _____ for a function, but is not part of the graph itself.



$f(x) = \frac{1}{x}, x \neq 0$

- In a reciprocal function:
  - there is a _____ asymptote that is the restriction for the denominator, or domain.
  - there is one _____ asymptote, k.
  - a _____. If $a = 0$, it cannot be a reciprocal function.
- If intercepts exist on the graph of a reciprocal function, then:
  - the x-intercepts, or zeros, of the graph are the values that make _____.
  - the y-intercept is the value for _____.

To continue, return to the Online Lesson.

Transforming Reciprocal Functions

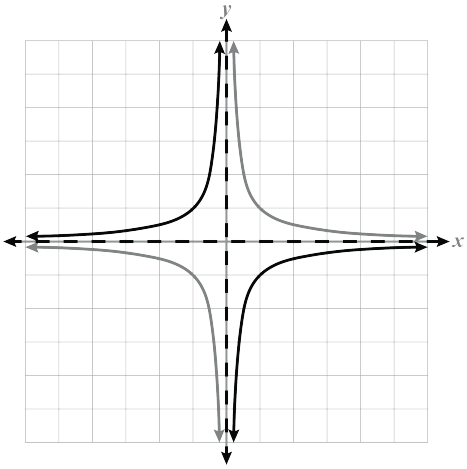
Fill in the guided notes as you watch the video in the Online Lesson.

- Recall from Algebra 1, the following is true of transformations of parabolas:
  - _____ reflects, stretches, and compresses a graph.
  - _____ shifts a graph left or right.
  - _____ shifts a graph up or down.
- Changes to a, h, and k are _____ on the reciprocal parent function across the coordinate plane.

Reflections, Stretches, and Compressions

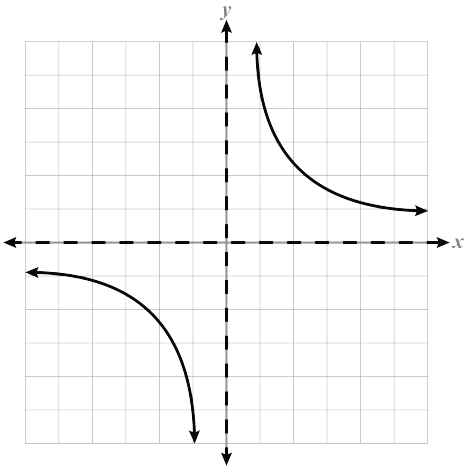
- The value of a will determine the location of the graph relative to the _____.
- The value of a also determines if the graph is a _____ and if it is stretched or compressed.
  - A reflection _____ the graph into the region opposite of the parent graph.
  - Stretching will move the graph _____ from the asymptotes.
  - Compression will move the graph _____ the asymptotes.
- If a is negative ($-a$) the graph is _____.

$$g(x) = -\frac{1}{x}, x \neq 0$$



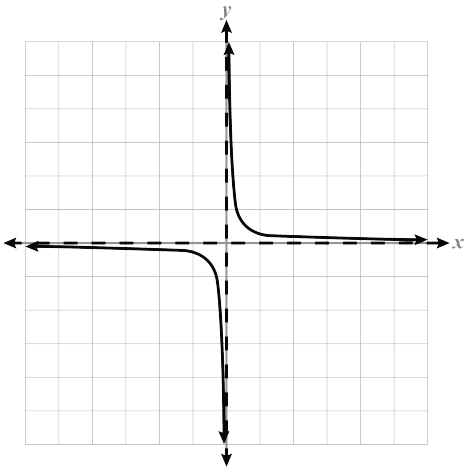
- If $|a| > 0$, the graph will _____ vertically.

$h(x) = \frac{5}{x}, x \neq 0$



- If $0 < |a| < 1$, the graph will _____ vertically.

$j(x) = \frac{1}{5x}, x \neq 0$



Horizontal and Vertical Shifts

- _____ moves the graph right ➔

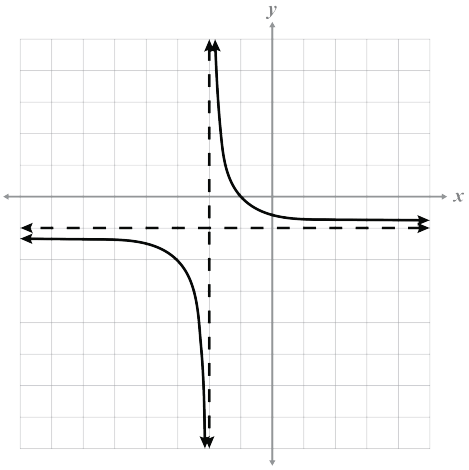
  (The h-value is positive)

- _____ moves the graph left ➔

  (The h-value is negative)

- _____ moves the graph up ⬆

- _____ moves the graph down ⬇



$g(x) = \frac{1}{x+2} - 1, x \neq -2$

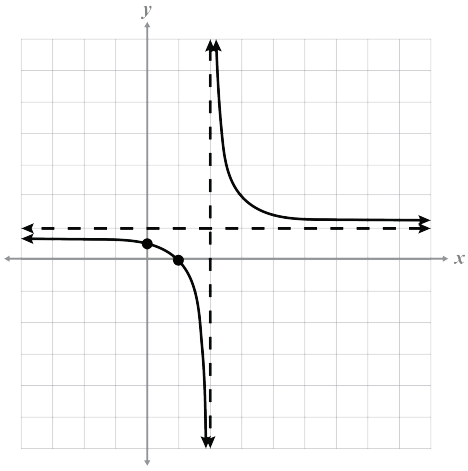
$a = 1, h = -2, k = -1$

Example 1

Complete the example as you watch the video in the Online Lesson.

Name any asymptotes and intercepts from the given graph. Describe the transformation from the parent graph.

$$g(x) = \frac{1}{x-2} + 1, x \neq 2$$



Plan

Name a, h, k
Name the equations for the asymptotes
Calculate the intercepts algebraically
Describe the transformation of the graph

Implement

Asymptotes: (read from the graph)

Intercepts: (estimate from graph, confirm algebraically)

x-intercept: y-intercept:

Some graphs will not have intercepts because the graph does not cross either axis.

Explain

$g(x)$ is two spaces right and one space up from the parent graph because

Example 2

Complete the example as you watch the video in the Online Lesson.

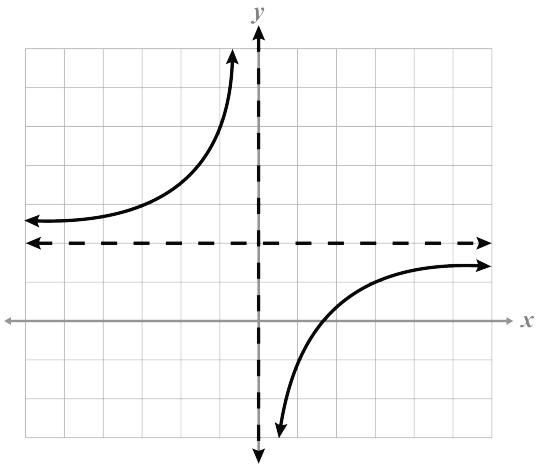
Name any asymptotes and intercepts from the given graph. Describe the transformation from the parent graph.

$$q(x) = -\frac{3}{x} + 2,\ x \neq 0$$

Asymptotes: (read from graph)

Intercepts: (estimate from graph, confirm algebraically)

x-intercept: y-intercept:



This graph is reflected and stretched vertically because $a = -3$. The graph is shifted up two spaces because $k = 2$.

Example 3

Complete the example as you watch the video in the Online Lesson.

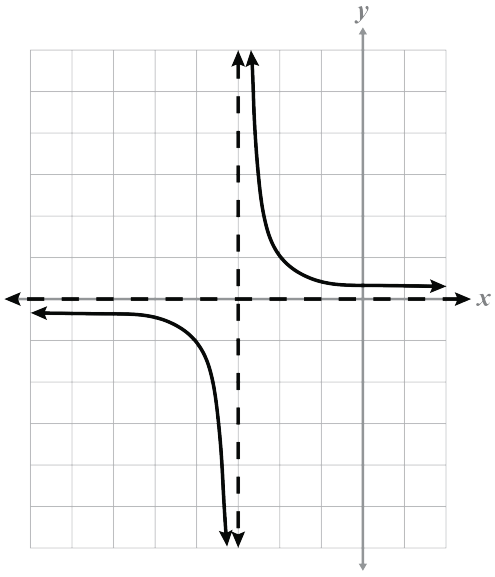
Name any asymptotes and intercepts from the given graph. Describe the transformation from the parent graph.

$$h(x) = \frac{1}{x+3},\ x \neq -3$$

Asymptotes:

Intercepts:

x-intercept: y-intercept:



Checkpoint: Reciprocal Functions

Name any asymptotes and intercepts from the equation. Describe the transformation from the parent graph.

$$f(x) = \frac{1}{x+5} - 4$$

To continue, return to the Online Lesson.

Domain and Range of Reciprocal Functions

Fill in the guided notes as you watch the video in the Online Lesson.

- When writing the domain and range for reciprocal functions, use _____ notation.
- This math shorthand allows you to _____ of numbers, rather than listing all of the values.

Symbol	Read as	Meaning
	set brackets, “the set of …”	a group or set
	“such that”	
	“an element of”	is part of a set
	“is not” (i.e. ≠: “is not equal to,” ∉: “is not an element of”)	symbol to show negation
	Possible sets of numbers	

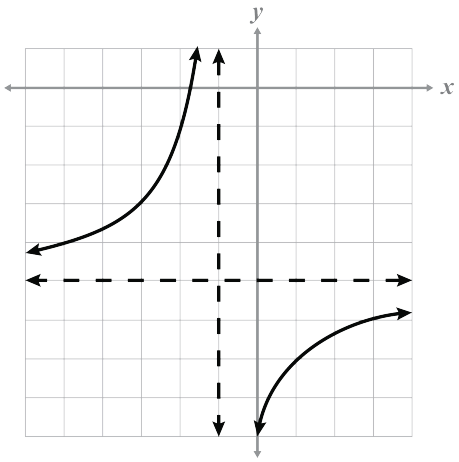
- Here are some examples using set-builder notation:
  - "The set of all x's such that x is an element of the Real number and cannot equal 2" _____
  - "The set of all q's such that q is an element of the Natural numbers, and between 2 and 8" This could also be written as $\{3, 4, 5, 6, 7\}$ _____
- The domain is the set of _____ for a function.
- The range is the set of _____ for a function.
- For reciprocal functions, use the _____ to help determine restrictions for the domain and range.
- Graphs do not cross vertical asymptotes because this would make the denominator _____.
- Vertical asymptotes are boundaries for the _____.
- Horizontal asymptotes show the end behavior of a hyperbola as the graph moves toward _____.

Example 4

Complete the example as you watch the video in the Online Lesson.

Name the domain and range of the reciprocal function.

$$f(x) = \frac{-4}{x+1} - 5$$



Plan

Identify a, h, k
Determine the domain and range in set-builder notation

Implement

Domain: $\{x \mid x \in \mathbb{R}, x \neq -1\}$

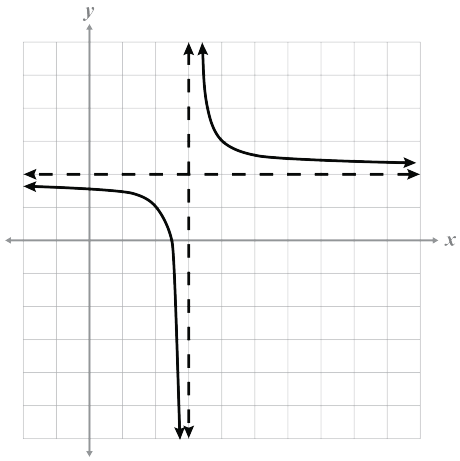
Range: $\{y \mid y \in \mathbb{R}, y \neq -5\}$

Example 5

Complete the example as you watch the video in the Online Lesson.

Name the domain and range of the reciprocal function.

$$y = \frac{1}{x-3} + 2$$



Checkpoint: Domain and Range of Reciprocal Functions

Name a, h, and k. Determine the domain and range of the reciprocal function.

$$g(x) = \frac{3}{x-11} + 7$$

To continue, return to the Online Lesson.

Graphing Reciprocal Functions

Fill in the guided notes as you watch the video in the Online Lesson.

- To graph a simple reciprocal function:

1) Find _____.

2) Plot the _____ and any _____ on the graph.

3) Sketch the hyperbola using the value of a to determine _____ and _____.

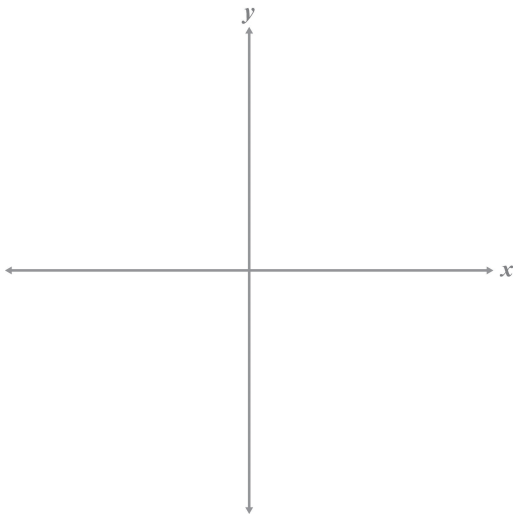
- Be sure to _____ of the graph because the coordinate planes are unscaled.
- The table of the parent function shows that many of the values are _____.
- Therefore, when graphing a reciprocal function, sketch the _____ rather than plot points.

Example 6

Complete the example as you watch the video in the Online Lesson.

Graph the asymptotes and intercepts. Sketch the hyperbola.

$$f(x) = -\frac{1}{x} + 3$$



Plan

Name a, h, k
Graph asymptotes
Solve for any intercepts
Graph intercepts
Sketch hyperbola

Implement

Intercepts:

$$0 = -\frac{1}{x} + 3$$

no y-intercept

$$-3 = -\frac{1}{x}$$

$$-3x = -1$$

$$x = \frac{1}{3}$$

Explain

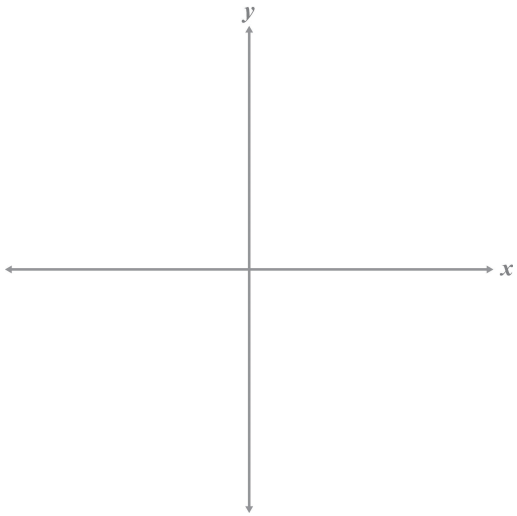
The hyperbola is a reflection because $a = -1$. The graph will shift up 3 spaces from the parent graph.

Example 7

Complete the example as you watch the video in the Online Lesson.

Graph the asymptotes and intercepts. Sketch the hyperbola.

$$g(x) = \frac{2}{x-4} - 2$$



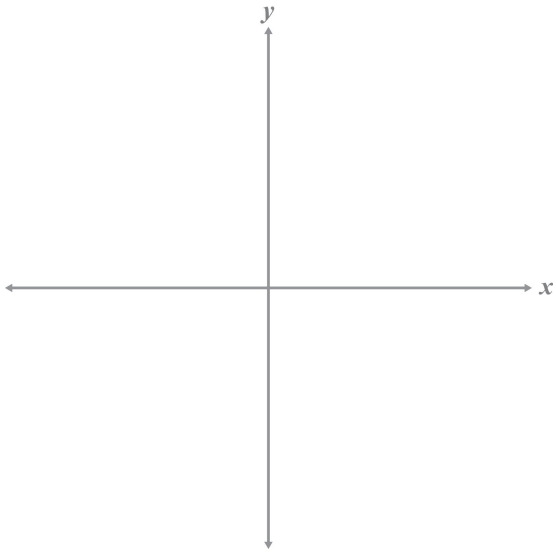
The hyperbola stretched, making it farther away from the asymptotes when $a = 2$. The values of h and k shift the function right 4, and down 2.

Example 8

Complete the example as you watch the video in the Online Lesson.

Graph the asymptotes and intercepts. Sketch the hyperbola.

$$f(x) = \frac{1}{3(x-1)}$$

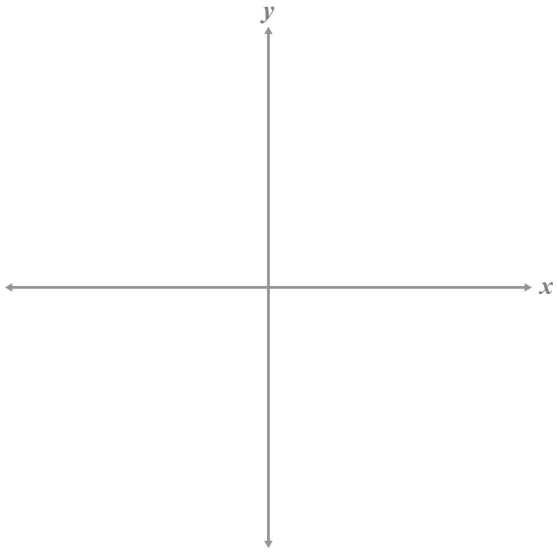


The hyperbola is closer to the asymptotes since the value of a is between 0 and 1. The graph shifts right one space.

Checkpoint: Graphing Reciprocal Functions

Graph the asymptotes and intercepts. Sketch the hyperbola.

$$y = \frac{-1}{x+3} - 1$$



To continue, return to the Online Lesson.

Practice 1

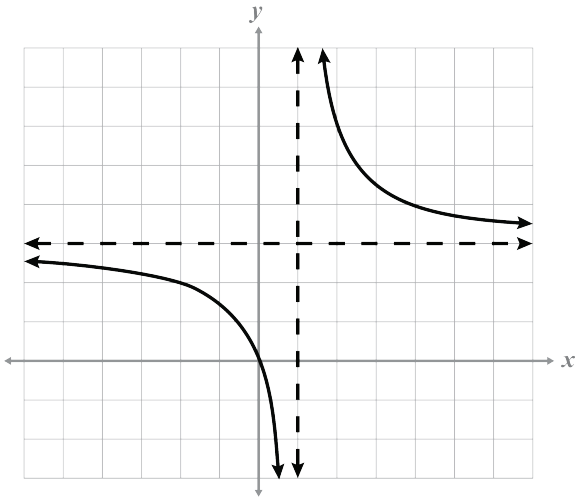
Complete problems on a separate sheet of paper.

1) Describe what the value of a does to the graph of a reciprocal function.

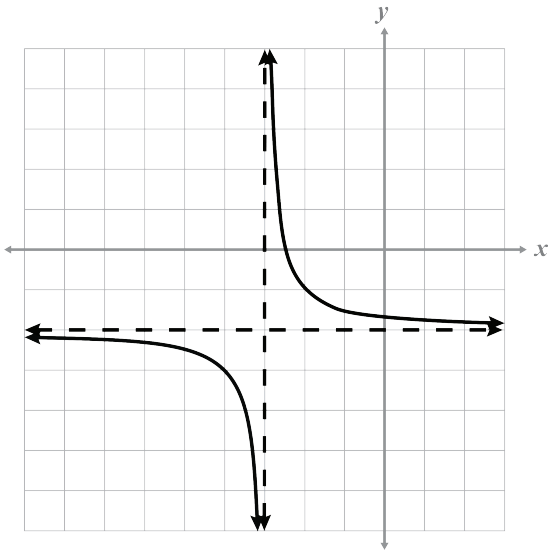
2) Describe what the value of k does to the graph of a reciprocal function.

Name any asymptotes and intercepts from the given graph. Describe the transformation from the parent graph.

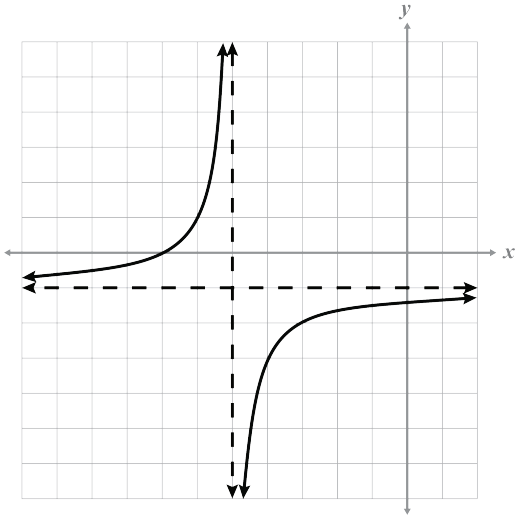
3) $g(x) = \dfrac{3}{x-1} + 3$



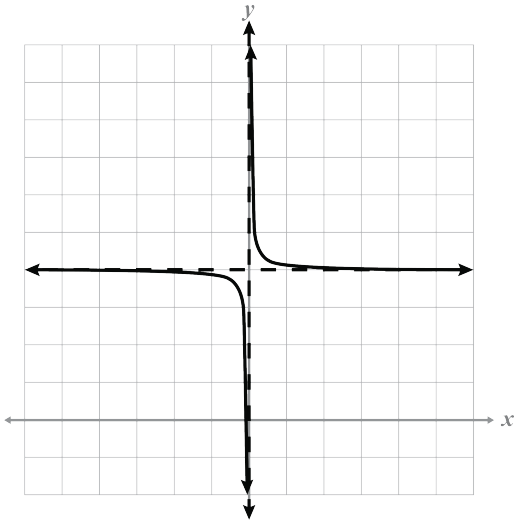
4) $f(x) = \dfrac{1}{x+3} - 2$



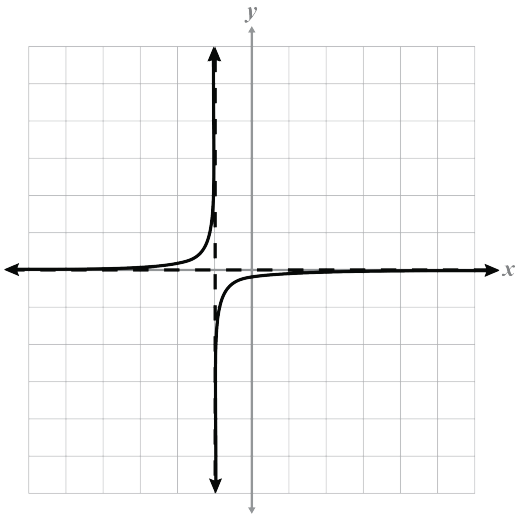
5) $h(x) = \dfrac{-2}{x+5} - 1$



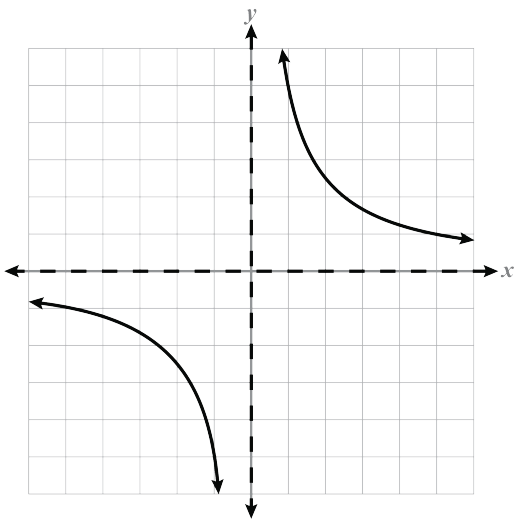
6) $f(x) = \dfrac{1}{8x} + 4$



7) $b(x) = \dfrac{-1}{6(x+1)}$



8) $j(x) = \dfrac{5}{x}$



Name a, h, and k. Determine the domain and range of the reciprocal function.

9) $f(x) = \dfrac{-1}{3x} + 6$

10) $h(x) = \dfrac{20}{x+4} - 5$

11) $n(x) = \dfrac{9}{x-12} + 14$

12) $a(x) = \dfrac{-1}{6(x-2)} + 4$

Graph the asymptotes and intercepts. Sketch the hyperbola.

13) $f(x) = \dfrac{4}{x+1} - 2$

14) $g(x) = \dfrac{-1}{x-4} + 1$

15) $h(x) = \dfrac{-1}{5x} - 3$

16) $f(x) = \dfrac{3}{x-1}$

17) $v(x) = \dfrac{1}{x} - 1$

18) $g(x) = \dfrac{-1}{2(x+2)} + 5$

To continue, return to the Online Lesson.

Mastery Check

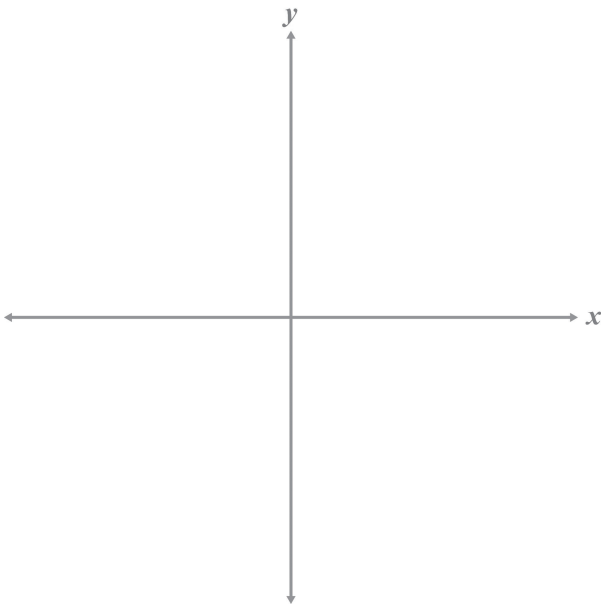
Show What You Know

A reciprocal function is stretched by 3 with the domain $\{x|x \in \mathbb{R}, x \neq -4\}$ and a range of $\{y|y \in \mathbb{R}, y \neq 2\}$.

A) Write an equation for a reciprocal function in the form:

$$y = \frac{1}{x-h} + k, x \neq h.$$

B) Graph the equation from part A. Label asymptotes and intercepts.



Use the numbers $\{-3, -2, -1, 0, 1, 2\}$ *only once* to create a reciprocal function with the following characteristics:

- Be a reflection with no stretch or compression
- The vertical and horizontal asymptotes must intersect in Quadrant IV

$$f(x) = \frac{\square}{x - \square} + \square$$

C) Write the equation of the reciprocal function.

D) Name the domain and range for the equation in part C.

Say What You Know

In your own words, talk about what you have learned using the objectives for this part of the lesson and your work on this page.

To continue, return to the Online Lesson.

Practice 2

Complete problems on a separate sheet of paper.

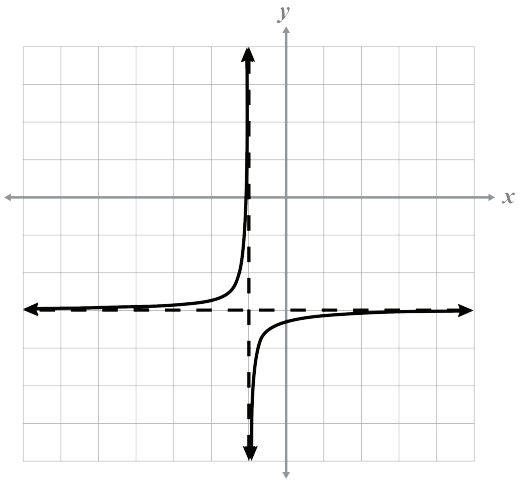
1) Which graph would be sketched closer to the origin? Explain.

$f(x) = \frac{1}{5x}$ $g(x) = \frac{1}{2x}$

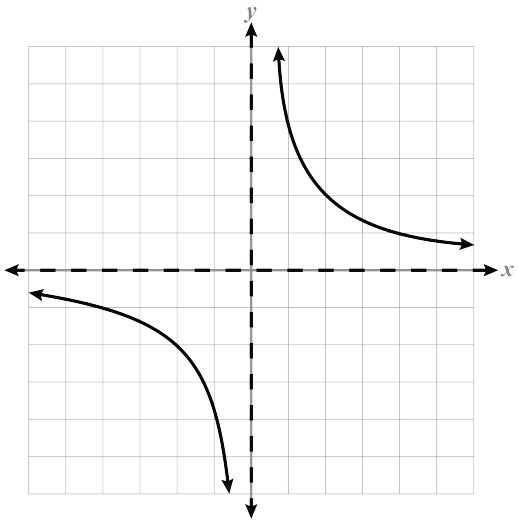
2) Describe what the value of h does to the graph of a reciprocal function.

Name any asymptotes and intercepts from the given graph. Describe the transformation from the parent graph.

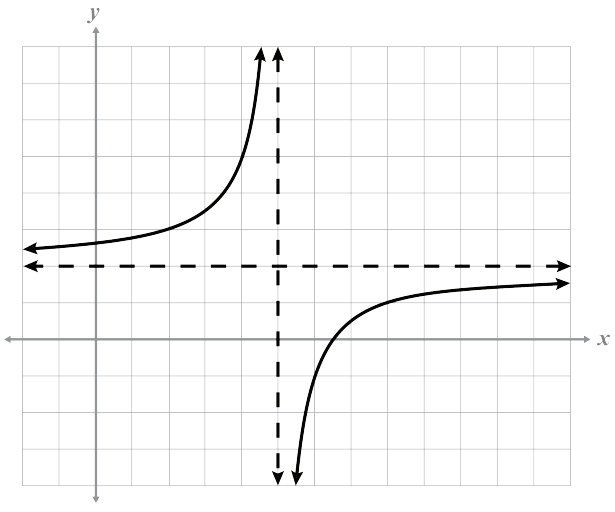
3) $f(x) = \frac{-1}{4(x+1)} - 3$



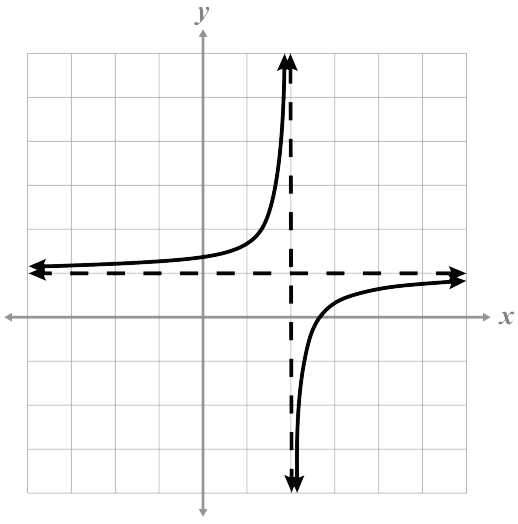
4) $g(x) = \frac{4}{x}$



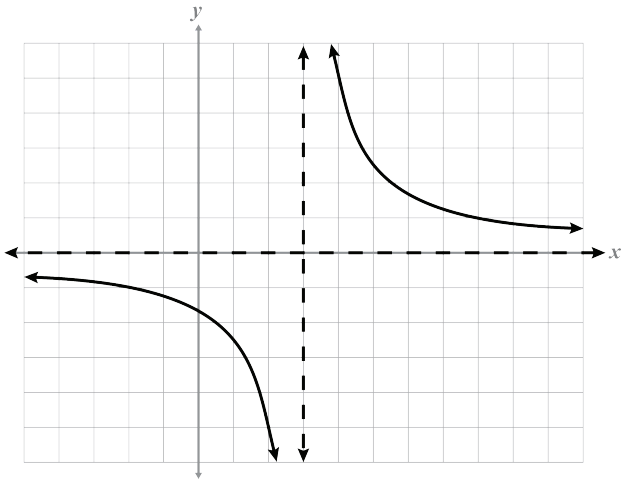
5) $h(x) = \frac{-3}{x-5} + 2$



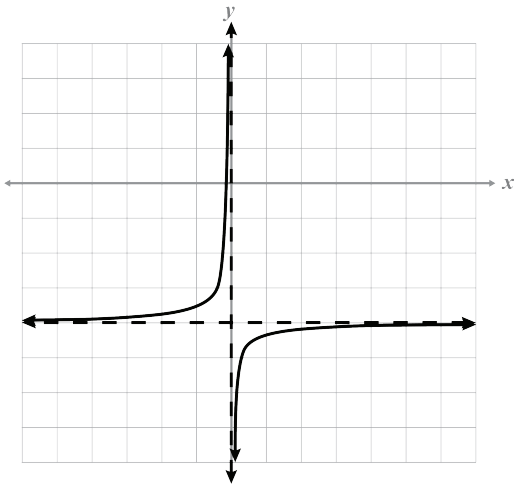
6) $j(x) = \frac{-2}{3(x-2)} + 1$



7) $b(x) = \dfrac{5}{x-3}$



8) $a(x) = \dfrac{-4}{9x} - 4$



Name a, h, and k. Determine the domain and range of the reciprocal function. Graph the asymptotes. Sketch the hyperbola.

9) $g(x) = \dfrac{6}{x+3} - 2$

10) $f(x) = \dfrac{-1}{5x} - 8$

11) $h(x) = \dfrac{4}{x-6} + 6$

12) $b(x) = \dfrac{-2}{x+4} + 5$

13) $a(x) = \dfrac{-5}{x-3}$

14) $k(x) = \dfrac{7}{x}$

To continue, return to the Online Lesson.