LESSON PRACTICE

12A

Find the roots, using the quadratic formula when necessary.

1. $X^2 + 6X + 2 = 0$

2. $X^2 - 5X + 4 = 0$

3. $3X^2 + 7X - 1 = 0$

4. $A^2 - 10A = 11$

5. $2Q^2 + 2 = 17Q$

6. $5X^2 + 15X + 10 = 0$

7. $1/4\ R^2 - 1/2\ R + 3/2 = 0$

8. $16X^2 = 2X + 4$

9. $2X^2 + 3X - 8 = 0$

10. $Y^2 = 3/4\ Y + 2$

LESSON PRACTICE

12B

Find the roots, using the quadratic formula when necessary.

1. $8X^2 - X - 3 = 0$

2. $7 = 2X^2 + X$

3. $Q^2 - 6Q + 3 = 0$

4. $2 + 3X + 4X^2 = 0$

5. $P = P^2 - 2$

6. $X^2 + 1/5\ X + 5 = 0$

7. $20X^2 + 40X = 30$

8. $5A^2 + 2A - 1 = 0$

9. $3X^2 = -5X$

10. $AX^2 + BX + C = 0$

SYSTEMATIC REVIEW

Find the roots, using the quadratic formula when necessary.

1. $X^2 - 5X + 6 = 0$

2. $X^2 + 4X + 2 = 0$

3. $X^2 - 3X + 1 = -6X$

4. $X^2 + 4X - 12 = 0$

5. $2X^2 + 2X + 5 = 0$

6. $X^2 + 8X = -16$

Complete the square.

7. $X^2 - 26X +$ _____

8. $2X^2 + 9X +$ _____

9. $X^2 +$ _____ $+ 400$

10. $X^2 -$ _____ $+ 14$

Solve for X. Complete the square when necessary.

11. $X^2 + 1/3\ X - 4/3 = 0$

12. Check the answers to #11 by placing them in the original equation.

13. Expand $(X - A)^6$.

14. What is the second term of $(1/2\ X - 3A)^4$?

15. Expand $(5 - 2A)^3$.

16. Find the cube root of $X^3 - 6X^2Y + 12XY^2 - 8Y^3$.

Put in standard form.

17. $\dfrac{6 + 5i}{3i - 2}$

18. $\dfrac{2 + \sqrt{-49}}{2 - \sqrt{-49}}$

Simplify, and combine like terms when possible.

19. $\dfrac{2}{3 - \sqrt{7}}$

20. $\dfrac{2 + \sqrt{5}}{2\sqrt{5} - 4}$

Find the roots, using the quadratic formula when necessary.

1. $2X^2 - 9X - 7 = 0$

2. $X^2 + 5X - 2 = 0$

3. $3X^2 + 7X + 4 = 0$

4. $X^2 - 6X + 12 = 0$

5. $5X^2 - 3X - 2 = 0$

6. $4X^2 + 1 = 4X$

Complete the square.

7. $X^2 + 5X +$ ____

8. $X^2 - 1/2\ X +$ ____

9. $25X^2 +$ ____ $+ 1$

10. $49X^2 -$ ____ $+ 4$

Solve for X. Complete the square when necessary.

11. $X^2 - 12X + 20 = 0$

12. Check the answers to #11 by placing them in the original equation.

13. Expand $(X + 1)^4$.

14. What is the fifth term of $(1/2\ X - 3A)^4$?

15. Expand $(10 - 1/X)^3$.

16. Find the cube root of $X^3 + 6X^2 + 12X + 8$.

Put in standard form.

17. $\dfrac{4 - 3i}{2i}$

18. $\dfrac{10 + \sqrt{-A}}{10 - \sqrt{-A}}$

Simplify, and combine like terms when possible.

19. $\dfrac{9}{7 + \sqrt{10}}$

20. $\dfrac{4 - \sqrt{6}}{3\sqrt{7} + 5}$

SYSTEMATIC REVIEW

12E

Find the roots, using the quadratic formula when necessary.

1. $X^2 + 2X - 8 = 0$
2. $X^2 - 6X = -8$
3. $2X^2 - 15X + 7 = 0$
4. $3X^2 + 4X = 7$
5. $2 = 5X + X^2$
6. $X^2 + 2X - 15 = 0$

Complete the square.

7. $4X^2 + 28X +$ _____
8. $9X^2 - 36X +$ _____
9. $36X^2 +$ _____ $+ 25$
10. $81X^2 -$ _____ $+ 121$

Solve for X. Complete the square when necessary.

11. $X^2 + 5X - 14 = 0$
12. Check the answers to #11 by placing them in the original equation.

13. Expand $(2X + 1)^5$.

14. What is the third term of $(1/3\ X + 2)^5$?

15. Expand $(X - 3/5)^3$.

16. Find the cube root of $8X^3 + 12X^2 + 6X + 1$.

Put in standard form.

17. $\dfrac{10 + i}{5i}$

18. $\dfrac{10}{5 - \sqrt{8}}$

Simplify, and combine like terms when possible.

19. $\dfrac{2 + 3\sqrt{6}}{1 - \sqrt{6}}$

20. $\dfrac{6 - \sqrt{2}}{10\sqrt{3} - 8}$

You have used the binomial theorem to find the terms when a binomial is raised to a power. Here is another method that uses factorials. It was discovered by a man named Leonard Euler in the 18th century.

The method is based on the version of Pascal's triangle shown below. If you remember that 0! equals one, you can reduce each fraction so that this becomes a regular Pascal's triangle. Also notice that this is similar to the triangle shown in lesson 10 in your instruction manual.

$$\frac{0!}{0!\,0!}$$

$$\frac{1!}{0!\,1!} \quad \frac{1!}{1!\,0!}$$

$$\frac{2!}{0!\,2!} \quad \frac{2!}{1!\,1!} \quad \frac{2!}{2!\,0!}$$

$$\frac{3!}{0!\,3!} \quad \frac{3!}{1!\,2!} \quad \frac{3!}{2!\,1!} \quad \frac{3!}{3!\,0!}$$

The notation $\binom{n}{r-1}$ is used in this new formula for terms of an expanded binomial. It is read as "n choose r −1."

The formula for the *r* term of $(a + b)^n$ is $\binom{n}{r-1} a^{n-r+1} b^{r-1}$. The letter *n* tells you what row of the triangle you are using, and *r* tells what term in that row is chosen. Remember that we start counting rows with 0. However, we can start counting terms with one because the formula has already subtracted one from *r*.

This is not as difficult as it looks! Study the examples, and compare what you are doing here to the method you have already learned.

Example 1

Find the second term of $(a + b)^3$ if n = 3 and r = 2.

Simplify terms.

$$\binom{n}{r-1} a^{n-r+1} b^{r-1} = \binom{3}{2-1} a^{3-2+1} b^{2-1} = \binom{3}{1} a^2 b^1$$

Change n to n!, or in this case, 3 to 3! Look at the second term (counting from one) of row 3 (counting from zero) to find the factorials for the denominator.

$$\frac{3!}{1!2!} a^2 b^1$$

Simplify.

$$\frac{3 \cdot \cancel{2!}}{1! \cdot \cancel{2!}} a^2 b^1 = 3a^2 b$$

Notice that the numbers in the factorial form of the denominator are the *b* and *a* exponents and that they add to produce the number in the numerator. This is always true, so you do not need to use the triangle.

Example 2

Find the third term of $(a + b)^6$ if n = 6 and r = 3.

Simplify terms.

$$\binom{n}{r-1} a^{n-r+1} b^{r-1} = \binom{6}{3-1} a^{6-3+1} b^{3-1} = \binom{6}{2} a^4 b^2$$

Change to factorials and simplify.

$$\frac{6!}{2!\;4!} a^4 b^2 = \frac{6 \cdot 5 \cdot \cancel{4!}}{2 \cdot \cancel{4!}} a^4 b^2 = 15a^4 b^2$$

Use factorials to find the requested term.

1. Find the fifth term of $(X + Y)^6$.

2. Find the second term of $(A + 2)^4$.

3. Find the third term of $(P + Q)^5$.

4. Find the fourth term of $(2X - 1)^7$.